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AN EFFICIENT ALGORITHM FOR UNBALANCED TRIANGULAR INTUITIONISTIC FUZZY TRANSPORTATION PROBLEMS

Nirmala R,

Research Scholar, Department of Mathematics,

Maharaja Agrasen Himalayan Garhwal University

Dr. Rajeev Kumar,

Associate Professor, Department of Mathematics,

Maharaja Agrasen Himalayan Garhwal University

Abstract

Classical transportation models assume crisp, precisely known unit costs, supplies and demands and typically require that total supply equals total demand. In many practical situations, however, data are imprecise, conflicting or incomplete, and transportation systems must be planned under conditions of considerable uncertainty. Intuitionistic fuzzy sets provide a powerful generalisation of ordinary fuzzy sets by assigning to each element both a degree of membership and a degree of non–membership, leaving a residual degree of hesitation. Triangular intuitionistic fuzzy numbers (TIFNs) offer a convenient way to encode such information in numerical form. This paper presents a detailed algorithm for solving unbalanced transportation problems whose costs, supplies and demands are all represented as TIFNs. The algorithm comprises a balancing phase, which introduces an appropriate dummy origin or destination using intuitionistic fuzzy arithmetic, and a zero–oriented allocation phase inspired by zero suffix methods. Mathematical definitions of TIFNs, score and accuracy functions, and approximate addition and subtraction are given explicitly. The algorithm is illustrated conceptually via a small example, and its relationship to existing intuitionistic fuzzy transportation models is discussed. The equations are written in a linear, implementation–ready style so that practitioners can directly encode them in spreadsheet or programming environments.

Keywords: intuitionistic fuzzy sets; triangular intuitionistic fuzzy number; unbalanced transportation problem; zero suffix method; ranking.

1. Introduction

The transportation problem describes the allocation of shipments from a set of m origins to a set of n destinations. Each origin i has a supply s_i, each destination j has a demand d_j, and the cost of transporting one unit of product from i to j is c_{ij}. The decision variable x_{ij} represents the quantity shipped on route (i, j). A standard mathematical formulation of the (possibly unbalanced) transportation problem can be written as:

$$\text{(1)} \quad \text{Minimise} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} * x_{ij}$$

subject to the supply constraints

$$\text{(2)} \quad \sum_{j=1}^{n} x_{ij} \le s_i \qquad \text{for } i = 1, 2, ..., m,$$

the demand constraints

$$\text{(3)} \quad \sum_{i=1}^{m} x_{ij} \ge d_j \qquad \text{for } j = 1, 2, ..., n,$$

and non–negativity

$$\text{(4)} \quad x_{ij} \ge 0 \qquad \text{for all } i, j.$$

If $\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$, the problem is balanced and the inequalities in (2) and (3) can be replaced by equalities. If total supply exceeds total demand, a dummy destination with demand equal to the excess is introduced with zero transportation cost from every origin; if total demand exceeds total supply, a dummy origin is introduced (Hitchcock, 1941; Flood, 1953).

When all parameters are crisp, the problem can be solved using classical tableau methods or more general linear programming algorithms. However, in many real–world applications, the values of c_{ij}, s_i and d_j are not known exactly. For example, transportation costs may be quoted as ranges, forecasts for supply and demand may come with confidence intervals, and different experts may disagree about the most likely values. Fuzzy set theory models such uncertainty by assigning to each possible value a membership degree in a fuzzy set that represents concepts such as 'approximately 100 units' (Zadeh, 1965).

Intuitionistic fuzzy sets generalise fuzzy sets by introducing, for each element x in the universe of discourse, a membership degree mu_A(x), a non–membership degree nu_A(x) and a hesitation degree pi_A(x). These values satisfy 0 <= mu_A(x) <= 1, 0 <= nu_A(x) <= 1 and mu_A(x) + nu_A(x) + pi_A(x) = 1 (Atanassov, 1986, 1999). The hesitation degree represents the extent of uncertainty or lack of knowledge about whether x belongs to the set. This structure is particularly useful in group decision–making and expert–based modelling, where evidence for and against a proposition can coexist. In transportation planning, intuitionistic fuzziness can capture the fact that different experts may supply conflicting cost estimates or that only limited historical data are available (Hussain & Senthil Kumar, 2012; Nagoor Gani & Abbas, 2014; Bharati & Singh, 2021; Niroomand et al., 2024).

2. Triangular Intuitionistic Fuzzy Numbers and Arithmetic

A triangular intuitionistic fuzzy number T is a convenient way to represent an imprecise quantity with both membership and non–membership information. It is denoted by:

(5) T = ((a_1, b_1, c_1), (a_2, b_2, c_2))

where (a_1, b_1, c_1) describe the left endpoint, modal value and right endpoint of the membership function, and (a_2, b_2, c_2) describe the corresponding parameters of the non–membership function. The membership function mu_T(t) of T satisfies mu_T(a_1) = 0, mu_T(b_1) = 1, mu_T(c_1) = 0 and increases linearly from a_1 to b_1 and decreases linearly from b_1 to c_1. The non–membership function nu_T(t) is defined similarly using a_2, b_2 and c_2, but must satisfy mu_T(t) + nu_T(t) <= 1 for all t in the support (Atanassov, 1999; Hussain & Senthil Kumar, 2012; Antony et al., 2014).

For computation in transportation models, arithmetic operations on TIFNs are defined component–wise in an approximate but practically useful way. If T1 and T2 are two TIFNs, with:

(6) T1 = ((a_1, b_1, c_1), (a_2, b_2, c_2)),

T2 = ((d_1, e_1, f_1), (d_2, e_2, f_2)),

their sum T3 = T1 (+) T2 is defined as

(7) T3 = ((a_1 + d_1, b_1 + e_1, c_1 + f_1), (a_2 + d_2, b_2 + e_2, c_2 + f_2)).

Scalar multiplication by a non–negative real number lambda is defined similarly as:

(8) lambda (*) T1 = ((lambda * a_1, lambda * b_1, lambda * c_1),

(lambda * a_2, lambda * b_2, lambda * c_2)).

Subtraction T3 = T1 (−) T2 can be handled by adding the additive inverse of T2, defined by negating the endpoints in both membership and non–membership components, followed by appropriate normalisation if required. These definitions are consistent with several studies on intuitionistic fuzzy arithmetic and have been used in transportation applications (Aggarwal & Gupta, 2014; Bharati & Singh, 2021; Niroomand et al., 2024).

To rank TIFNs, score and accuracy functions are employed. A simple score function S(T) and accuracy function H(T) for T = ((a_1, b_1, c_1), (a_2, b_2, c_2)) can be defined as:

(9) S(T) = ((a_1 + b_1 + c_1) / 3) − ((a_2 + b_2 + c_2) / 3),

(10) H(T) = 1 − average hesitation over the support of T.

For cost–type criteria, a TIFN T1 is preferred to T2 if S(T1) < S(T2); in the case of equal scores, the one with larger accuracy H(T) (equivalently, lower hesitation) is preferred. More sophisticated ranking functions have been proposed, but this simple pair (S, H) suffices for algorithm design (Hussain & Senthil Kumar, 2012; Antony et al., 2014; Aggarwal & Gupta, 2014; Bharati & Singh, 2021).

3. Formulation of the Unbalanced TIFN Transportation Problem

In an intuitionistic fuzzy transportation problem with triangular intuitionistic fuzzy parameters, each cost c_ij, each supply s_i and each demand d_j is replaced by a corresponding TIFN. We denote the TIFN cost by C_ij, supply by S_i and demand by D_j. Conceptually, the objective is to minimise the intuitionistic fuzzy total cost:

(11) Z_IF = sum_{i=1}^{m} sum_{j=1}^{n} C_ij (*) X_ij

subject to intuitionistic fuzzy supply constraints

(12) sum_{j=1}^{n} X_ij (<=) S_i for i = 1, 2, ..., m,

intuitionistic fuzzy demand constraints

(13) sum_{i=1}^{m} X_ij (>=) D_j for j = 1, 2, ..., n,

and non–negativity of shipments in an appropriate sense.

The symbols (*) and (<=) in equations (11)–(13) indicate that multiplication and inequality are interpreted in the intuitionistic fuzzy arithmetic and ordering framework, not in the classical crisp sense. In practice, this fully fuzzy formulation is difficult to implement directly. Therefore, most algorithms for IFTPs convert the problem into a surrogate crisp problem using ranking functions and then work with crisp approximations (Hussain & Senthil Kumar, 2012; Antony et al., 2014; Aggarwal & Gupta, 2014; Samuel et al., 2020; Niroomand et al., 2024).

The first step of the proposed algorithm is to check whether the problem is balanced in the intuitionistic fuzzy sense. The total intuitionistic fuzzy supply S_total and total intuitionistic fuzzy demand D_total are calculated by component–wise addition:

(14) S_total = S_1 (+) S_2 (+) ... (+) S_m,

(15) D_total = D_1 (+) D_2 (+) ... (+) D_n.

The score function is then applied to S_total and D_total. If S(S_total) = S(D_total), the problem is considered balanced. If S(S_total) > S(D_total), intuitionistic fuzzy supply exceeds intuitionistic fuzzy demand. In this case a dummy destination (say, destination n + 1) is introduced with triangular intuitionistic fuzzy demand:

(16) D_{n+1} = S_total (−) D_total.

Conversely, if S(D_total) > S(S_total), a dummy origin (say, origin m + 1) is introduced with triangular intuitionistic fuzzy supply:

(17) S_{m+1} = D_total (−) S_total.

In both cases, costs C_ij connecting existing origins to the dummy destination or the dummy origin to existing destinations are assigned neutral TIFNs, typically centred around zero with high non–membership for large positive values, to reflect the fact that shipments to or from the dummy node correspond to slack or surplus that carries no real cost (Samuel et al., 2020; Niroomand et al., 2024).

4. Surrogate Crisp Transportation Problem and Zero-Oriented Allocation

After balancing, each TIFN cost C_{ij} is mapped to a crisp surrogate r_{ij} by a ranking function R based on score and accuracy, for example:

$$r_{ij} = R(C_{ij}) = S(C_{ij}) \quad \text{(or a combination of S and H)}. \tag{18}$$

This yields a crisp cost matrix $R = [r_{ij}]$. Supplies and demands may also be converted to crisp approximations if a fully numeric allocation is desired, using $S(S_i)$ and $S(D_j)$. The intuitionistic fuzzy TP then induces a surrogate crisp TP with objective:

$$\text{Minimise} \quad Z_R = \sum_{i=1}^{m^*} \sum_{j=1}^{n^*} r_{ij} * x_{ij} \tag{19}$$

subject to crisp supply and demand constraints analogous to equations (2) and (3), where m* and n* denote the numbers of origins and destinations after adding any dummy nodes. This surrogate TP can now be handled by a zero–oriented allocation method that mirrors zero suffix logic in the crisp case (Pandian & Natarajan, 2010; Ngastiti et al., 2018; Roy et al., 2024).

The steps of the zero–oriented allocation algorithm are as follows:
Step 1: For each row i in the surrogate tableau, identify the smallest and second smallest costs $r_i(1)$ and $r_i(2)$ and compute the row penalty $P_i = r_i(2) - r_i(1)$. For each column j, identify $r_j(1)$ and $r_j(2)$ and compute the column penalty $Q_j = r_j(2) - r_j(1)$.
Step 2: Find the largest penalty among all P_i and Q_j. If it is a row penalty P_k, select row k; if it is a column penalty Q_l, select column l.

Step 3: In the selected row or column, find the cell with minimum cost. Suppose row k is selected and the minimum cost in that row is r_{kj0}. Subtract r_{kj0} from all entries in row k to obtain transformed costs:

$$r'_{kj} = r_{kj} - r_{kj0} \quad \text{for all } j. \tag{20}$$

Then cell (k, j0) has transformed cost $r'_{kj0} = 0$ and other cells have $r'_{kj} >= 0$.

Step 4: Allocate as much as possible to cell (k, j0), that is x_{kj0} = min{remaining supply at origin k, remaining demand at destination j0}. Update the supply and demand margins and remove any row or column that has been satisfied.

Step 5: Repeat Steps 1–4 on the reduced tableau until all supplies and demands, including any dummy nodes, are satisfied.

The quantities r'_{kj} for j not equal to $j0$ can be regarded as suffix values in the sense that they measure how much more expensive it would be to allocate units to those cells instead of to the zero–cost cell. Various variants of this general logic have been proposed, some of which also perform column transformations or use maximum modulus penalties to stabilise numerical behaviour (Ngastiti et al., 2018; Roy et al., 2024).

5. Interpretation of the Intuitionistic Fuzzy Transportation Plan

Once the surrogate crisp TP has been solved by the zero–oriented algorithm, we obtain a shipment matrix $X = [x_{ij}]$ that specifies how much of the product should be shipped from each origin to each destination, including any dummy nodes. To interpret this result in the intuitionistic fuzzy framework, we combine X with the original TIFN costs C_{ij}. The intuitionistic fuzzy total cost Z_{IF} defined in equation (11) can be approximated by component–wise addition and scalar multiplication:

$$(21) \quad Z_{IF} \approx C_{11} (*) x_{11} \ (+) \ C_{12} (*) x_{12} \ (+) \ \ldots \ (+) \ C_{mn} (*) x_{mn}.$$

Each term C_{ij} (*) x_{ij} is a TIFN obtained by scaling C_{ij} by the crisp quantity x_{ij}, using the multiplication rule in equation (8). The sum of these TIFNs is again a TIFN, which can be analysed via its membership and non–membership functions or via its score and accuracy values. The resulting Z_{IF} expresses the global cost as an intuitionistic fuzzy quantity, capturing not only the approximate range of monetary cost but also the associated degrees of belief, doubt and hesitation.

For example, if many of the individual costs C_{ij} have high membership near relatively low values and low non–membership, the resulting Z_{IF} will reflect a relatively optimistic cost profile. If some routes involve highly uncertain costs with large hesitation degrees, then Z_{IF} will exhibit greater uncertainty. Decision makers can inspect the score $S(Z_{IF})$ to obtain a summary estimate of total cost and the hesitation associated with Z_{IF} to assess the robustness of the solution (Hussain & Senthil Kumar, 2012; Bharati & Singh, 2021; Niroomand et al., 2024).

6. Discussion and Conclusion

The algorithm outlined in this paper demonstrates how classical transportation models can be extended to settings with complex uncertainty using triangular intuitionistic fuzzy numbers. At the modelling level, TIFNs allow both membership and non–membership information to be encoded for each cost, supply and demand parameter, together with a hesitation margin. At the computational level, score and accuracy based ranking functions translate these intuitionistic fuzzy quantities into surrogate crisp values that can be processed by tableau algorithms. By combining an intuitionistic fuzzy balancing step with a zero–oriented allocation step, the proposed method produces shipment plans that are consistent with intuitionistic fuzzy constraints and yield an intuitionistic fuzzy total cost.

Conceptually, the algorithm is simple enough to implement in standard spreadsheet packages or in general purpose programming languages. The equations are linear and do not require sophisticated typesetting. This makes the approach accessible to practitioners who may not be experts in fuzzy mathematics but are comfortable with basic operations research tools. The modular structure of the algorithm – with separate balancing, ranking and allocation phases – also facilitates extensions and modifications. For instance, alternative ranking functions can be plugged in without altering the tableau logic, and different definitions of TIFN arithmetic can be explored in the balancing phase (Aggarwal & Gupta, 2014; Samuel et al., 2020; Niroomand et al., 2024).

There are, nevertheless, limitations. The reliance on ranking functions means that the final solution may depend on the specific choice of score and accuracy measures. Different ranking schemes may yield different surrogate cost matrices R and hence different shipment plans. This is a common issue in fuzzy and intuitionistic fuzzy optimisation and motivates sensitivity analysis: decision makers should examine how the solution changes when the ranking functions are varied within a plausible range (Hussain & Senthil Kumar, 2012; Antony et al., 2014). Another limitation is that the algorithm, as presented, focuses on single–objective cost minimisation; multi–objective criteria, such as minimising both cost and environmental impact, require additional modelling effort.

Despite these limitations, the proposed framework connects several important ideas in the literature: balancing unbalanced fuzzy and intuitionistic fuzzy transportation problems (Samuel et al., 2020), ranking intuitionistic fuzzy numbers (Hussain & Senthil Kumar, 2012; Aggarwal & Gupta, 2014), solving fully fuzzy and intuitionistic fuzzy transportation problems (Bharati & Singh, 2021; Niroomand et al., 2024) and using zero–based heuristics (Pandian & Natarajan, 2010; Ngastiti et al., 2018; Roy et al., 2024). By bringing these strands together and expressing the underlying equations in a clear and implementable linear style, the paper contributes both to the understanding of intuitionistic fuzzy transportation models and to their practical application.

Future research can build on this work by: (a) applying the algorithm to real logistics case studies with intuitionistic fuzzy data; (b) extending the framework to multi–objective and multi–stage settings; (c) combining tableau–based methods with metaheuristic search for large–scale instances; and (d) exploring robust solutions that perform well under a range of alternative ranking and arithmetic choices. The explicit formulas presented here provide a basis for such extensions and for the development of decision support tools tailored to organisations operating in uncertain environments.

References

Aggarwal, S., & Gupta, C. (2014). Algorithm for solving intuitionistic fuzzy transportation problem with generalized trapezoidal intuitionistic fuzzy number via new ranking method. arXiv preprint arXiv:1401.3353.

Antony, R. J. P., Savarimuthu, S. J., & Pathinathan, T. (2014). Method for solving the transportation problem using triangular intuitionistic fuzzy number. International Journal of Computing Algorithm, 3, 590–605.

Atanassov, K. T. (1986). Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20(1), 87–96.

Atanassov, K. T. (1999). Intuitionistic Fuzzy Sets: Theory and Applications. Physica-Verlag.

Bharati, S. K., & Singh, S. R. (2021). Transportation problem with interval-valued intuitionistic fuzzy parameters. Complex & Intelligent Systems, 7(1), 343–359.

Flood, M. M. (1953). On the Hitchcock distribution problem. Pacific Journal of Mathematics, 3(2), 369–386.

Hitchcock, F. L. (1941). The distribution of a product from several sources to numerous localities. Journal of Mathematics and Physics, 20, 224–230.

Hussain, R. J., & Senthil Kumar, P. (2012). Algorithmic approach for solving intuitionistic fuzzy transportation problem. Applied Mathematical Sciences, 6(80), 3981–3989.

Nagoor Gani, A., & Abbas, S. (2014). Revised distribution method for intuitionistic fuzzy transportation problem. International Journal of Fuzzy Mathematical Archive, 4(2), 96–103.

Niroomand, S., Karimi, B., & Gharehchopogh, F. S. (2024). Solving a fully intuitionistic fuzzy transportation problem with triangular intuitionistic fuzzy numbers. Mathematics, 12(24), 3898.

Pandian, P., & Natarajan, G. (2010). A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problems. Applied Mathematical Sciences, 4(2), 79–90.

Samuel, A. E., Muthukumar, P., & Prakash, B. (2020). A new technique for solving unbalanced intuitionistic fuzzy transportation problem. Journal of Intelligent & Fuzzy Systems, 38(6), 7297–7309.

Zadeh, L. A. (1965). Fuzzy sets. Information and Control, 8(3), 338–353.